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## CONTACT PROBLEM FOR A STAMP WITH A RECTANGULAR BASE

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A method is proposed to solve the problem of impressing a rectangular stamp with arbitrary ratio between the sides into an elastic isotropic half-space, based on reduction of the problem to two-dimensional dual integral equations. A method of reducing these equations to an infinite system of linear algebraic equations is indicated. Formulas are obtained to determine the pressure on the contact area and the displacement of the stamp.

The papers [1 — 4] have been devoted to contact problems for a rectangular stamp. The problem of impressing a stamp with a base in the form of a narrow rectangle into an elastic half-space has been studied in [5 — 7].

The method of solution used in this paper is a further development and extension (to the case of two-dimensional dual equations) of the method used in [7].

**1.** We use a rectangular $x$, $y$, $z$ coordinate system whose $z$-axis is perpendicular to the boundary of the half-space. Let a stamp with a rectangular planform (Fig. 1) be

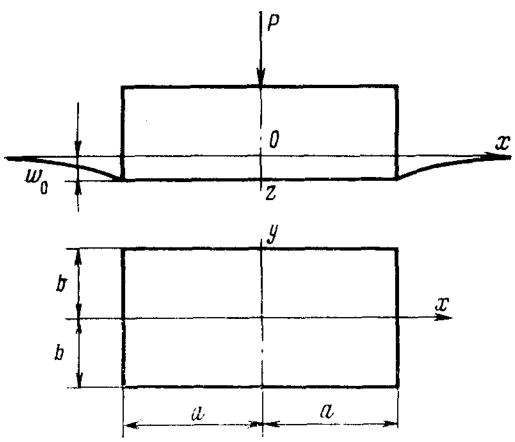


Fig. 1

impressed in an elastic isotropic half-space $z \geqslant 0$ under the effect of a vertical force $P$ directed along the $z$-axis and passing through the center of gravity of the stamp. We assume that the stamp with a flat base, is displaced strictly translationally in the direction of the $z$-axis and we consider no friction to originate between the stamp and the half-space and there is no load on the half-space outside the contact area.

The boundary conditions at $z = 0$ are in this case

$$
\begin{aligned}
&w(x, y, 0) = w_0 \qquad (|x| \leqslant a, \ |y| \leqslant b)\\
&\sigma_z(x, y, 0) = 0 \qquad (|x| > a, \ |y| > b)\\
&\tau_{xz}(x, y, 0) = \tau_{yz}(x, y, 0) = 0 \quad (-\infty < x, y < \infty)
\end{aligned}
$$

($w$ is the projection of the displacement vector on the $z$-axis).

Applying a two-dimensional Fourier integral transform to the Lamé equilibrium equations in rectangular coordinates, we find

$$
w(x, y, 0) = \frac{1-\nu^2}{\pi E} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (\alpha^2 + \beta^2)^{-1/2} p^{**}(\alpha, \beta)\, e^{-i(\alpha x + \beta y)} d\alpha d\beta \tag{1.2}
$$

$$
p(x, y) = -\sigma_z(x, y, 0)
$$

($p^{**}$ is the two-dimensional Fourier transform of the reaction pressure $p$). Formula (1.2) is valid under the condition that there are no shear stresses $\tau_{xz}$ and $\tau_{yz}$ at $z = 0$). On the basis of the inversion formulas for the two-dimensional Fourier transform, we have

$$
p(x, y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p^{**}(\alpha, \beta)\, e^{-i(\alpha x + \beta y)} d\alpha d\beta \tag{1.3}
$$

Satisfying the boundary conditions (1.1) and using (1.2) and (1.3), we arrive at two-dimensional dual integral equations

$$
\int_0^{\infty} \int_0^{\infty} \frac{p^{**}(\alpha, \beta)}{(\alpha^2+\beta^2)^{1/2}} \cos x\alpha \cos y\beta\, d\alpha d\beta = c \qquad (0 \leqslant x \leqslant a, \ \ 0 \leqslant y \leqslant b) \tag{1.4}
$$

$$
\int_0^{\infty} \int_0^{\infty} p^{**}(\alpha, \beta) \cos x\alpha \cos y\beta\, d\alpha d\beta = 0 \qquad (a < x < \infty, \ \ b < y < \infty)
$$

$$
c = \frac{\pi E w_0}{4(1-\nu^2)}
$$

The symmetric conditions of the problem relative to the $x$- and $y$-axes have also been taken into account in forming the dual equations (1.4). By solving (1.4), the function

$p^{**}(\alpha, \beta)$ can be found, and then by using (1.3) we obtain the distribution of the reaction pressure $p(x, y)$ over the contact area which is of interest to us.

**2.** Let us seek the solution of the dual integral equations (1.4) in the form

$$p^{**}(\alpha, \beta) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} B_{mn} \frac{J_{\lambda+2m}(a\alpha)\, J_{\mu+2n}(b\beta)}{(2\alpha)^{\lambda}(2\beta)^{\mu}} \qquad \left(\lambda, \mu > -\frac{1}{2}\right) \tag{2.1}$$

where $J_n(x)$ is the Bessel function of the first kind. Furthermore, it is known [8] that

$$\int_0^{\infty} x^{-\nu} J_{\nu+2n}(ax)\cos(yx)\,dx = \tag{2.2}$$

$$\begin{cases} (-1)^n 2^{\nu-1} a^{-\nu}(2n)!\,\Gamma(\nu)\,[\Gamma(2\nu+2n)]^{-1}(a^2-y^2)^{\nu-1/2}\, C_{2n}^{\nu}(y/a) & (0<y<a) \\ 0 \quad (a<y<\infty) \end{cases}$$

$$\operatorname{Re}\nu > -{}^1/_2, \quad a > 0$$

($C_n^{\nu}(x)$ is a Gegenbauer polynomial). Substituting (2.1) into the second equation in (1.4) and taking (2.2) into account, we see that the second equation in (1.4) is satisfied. Then, substituting (2.1) into the first equation in (1.4), we obtain

$$\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} B_{nm} \int_0^{\infty}\int_0^{\infty} \frac{J_{\lambda+2m}(a\alpha)\, J_{\mu+2n}(b\beta)}{(2\alpha)^{\lambda}(2\beta)^{\mu}(\alpha^2+\beta^2)^{1/2}} \cos x\alpha \cos y\beta\, d\alpha\, d\beta = c \tag{2.3}$$

$$(0 \leqslant x \leqslant a, \quad 0 \leqslant y \leqslant b)$$

It can be shown that

$$\cos x\alpha \cos y\beta = 2^{2(\lambda+\mu)}\Gamma(\lambda)\,\Gamma(\mu)\, a^{-\lambda} b^{-\mu} \times$$

$$\sum_{i=0}^{\infty}\sum_{k=0}^{\infty} (-1)^{i+k}(\lambda+2i)(\mu+2k)\, C_{2i}^{\lambda}\left(\frac{x}{a}\right) C_{2k}^{\mu}\left(\frac{y}{b}\right) \times$$

$$\frac{J_{\lambda+2i}(a\alpha)\, J_{\mu+2k}(b\beta)}{(2\alpha)^{\lambda}(2\beta)^{\mu}}$$

By using this expression, (2.3) becomes

$$\sum_{m=0}^{\infty}\sum_{k=0}^{\infty} E_{ik} C_{2i}^{\lambda}\left(\frac{x}{a}\right) C_{2k}^{\mu}\left(\frac{y}{b}\right) = 1 \qquad (0 \leqslant x \leqslant a, \quad 0 \leqslant y \leqslant b) \tag{2.4}$$

$$E_{ik} = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} B_{mn}^{(1)} D_{mnik}^{(1)}$$

$$B_{mn}^{1} = 2^{2(\lambda+\mu)}\Gamma(\lambda)\,\Gamma(\mu)\, a^{-\lambda} b^{-\mu} c^{-1} B_{mn}$$

$$D_{mnik}^{(1)} = (-1)^{i+k}(\lambda+2i)(\mu+2k) \times$$

$$\int_0^{\infty}\int_0^{\infty} \frac{J_{\lambda+2m}(a\alpha)\, J_{\mu+2n}(b\beta)\, J_{\lambda+2i}(a\alpha)\, J_{\mu+2k}(b\beta)}{(2\alpha)^{2\lambda}(2\beta)^{2\mu}(\alpha^2+\beta^2)^{1/2}}\, d\alpha\, d\beta$$

Expanding the right side in the first formula of (2.4) in Gegenbauer polynomials, we find

$$E_{ik} = \begin{cases} 1 & \text{for} \quad i = k = 0 \\ 0 & \text{for} \quad i \neq 0 \quad \text{or} \quad k \neq 0 \end{cases} \tag{2.5}$$

We set

$$X_{mn} = a^{2\lambda-1}b^{2\mu} B_{mn}^{(1)}, \quad D_{mnik}^{(1)} = a^{2\lambda-1}b^{2\mu} D_{mnik} \tag{2.6}$$

Then the second expression in (2.4) becomes

$$\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} D_{mnik} X_{mn} = E_{ik} \qquad (ik = 0, 1, 2, \ldots) \tag{2.7}$$

Therefore, an infinite system of linear algebraic equations has been obtained. Solving this system (2.7), we can find the coefficients $X_{mn}$. It is apparently impossible to find the exact solution of the infinite system of equations (2.7). However, an approximate solution of this system can be obtained by using the method of reduction or by successive approximations.

Substituting the expression for $p^{**}(\alpha, \beta)$ from (2.1) into the inversion formula for the two-dimensional Fourier cosine transform and taking account of (2.2), we find

$$p(x, y) = p_1(x, y) p_2(x, y) \qquad (0 \leqslant x < a, \quad 0 \leqslant y < b) \tag{2.8}$$

$$p_1(x, y) = \left(1 - \frac{x^2}{a^2}\right)^{\lambda-1/2} \left(1 - \frac{y^2}{b^2}\right)^{\mu-1/2}$$

$$p_2(x, y) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} A_{mn} C_{2m}^{\lambda} \left(\frac{x}{a}\right) C_{2n}^{\mu} \left(\frac{y}{b}\right)$$

$$p(x, y) = 0 \qquad (a < x < \infty, \quad b < y < \infty) \tag{2.9}$$

$$B_{mn} = (-1)^{m+n} 2\pi a^{1-\lambda} b^{1-\mu} \frac{\Gamma(2\lambda + 2m) \Gamma(2\mu + 2n)}{(2m)! (2n)! \Gamma(\lambda) \Gamma(\mu)} A_{mn} \tag{2.10}$$

Formula (2.8) establishes the pressure distribution law $p(x, y)$ over the contact area between a rectangular stamp and an elastic isotropic half-space. The force acting on the stamp is

$$P = \int_{-a}^{a} dx \int_{-b}^{b} p(x, y)\, dy = 2^{2(1-\lambda-\mu)} \pi^2 ab A_{00} \frac{\Gamma(2\lambda) \Gamma(2\mu)}{\lambda\mu [\Gamma(\lambda) \Gamma(\mu)]^2} \tag{2.11}$$

Relationships from [9] were used in deducing this formula.

**3.** By using (2.4), (2.6), (2.8), we find

$$A_{mn} = (-1)^{m+n} \frac{c\,(2m)!\,(2n)!}{2^{1+2\lambda+2\mu} \pi b \Gamma(2\lambda + 2m) \Gamma(2\mu + 2n)} X_{mn}, \quad m, n = 0, 1, \ldots \tag{3.1}$$

Knowing the coefficients $A_{mn}$, the pressure $p(x, y)$ can be determined at any point of the contact area by means of (2.8). The coefficients $A_{mn}$ are related to the coefficients $X_{mn}$ by formula (3.1), they can be found from the system of equations (2.7).

The quantities $D_{mnik}$ enter into the system (2.7). By using (2.4) and (2.6) and by introducing new variables $x = a\alpha$, $y = b\beta$, we have

$$D_{mnik} = (-1)^{i+k} (\lambda + 2i)(\mu + 2k) \times \tag{3.2}$$

$$\int_0^{\infty}\int_0^{\infty} \frac{J_{\lambda+2m}(x) J_{\lambda+2i}(x) J_{\mu+2n}(y) J_{\mu+2k}(y)}{(2x)^{2\lambda} (2y)^{2\mu} (\varepsilon^2 x^2 + y^2)^{1/2}} dx dy \quad (\varepsilon = b / a)$$

Let us determine the depth of impression $w_0$ of the stamp under the effect of the force $P$. By using the expression for $c$ in (1.4), and (2.11) and (3.1), we finally find

$$w_0 = 2^{1+4(\lambda+\mu)} \frac{\lambda\mu\,[\Gamma(\lambda)\,\Gamma(\mu)]^2}{\pi^2 X_{00}} \frac{P(1-\nu^2)}{Ea} \tag{3.3}$$

It is interesting to calculate the pressure $p$ for $x = 0$ and $y = 0$ (under the center of the stamp). We have

$$p(0,0) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} A_{mn} C_{2m}^{\lambda}(0)\, C_{2n}^{\mu}(0)$$

Using the result from [10]

$$C_n^{\lambda}(0) = \begin{cases} 0, & \text{if } n \text{ is odd} \\ (-1)^m \dfrac{(\lambda)_m}{m!}, & \text{if } n = 2m \text{ is even} \end{cases}$$

($(\lambda)_m = \Gamma(\lambda + m) / \Gamma(\lambda)$ is the Pochhammer symbol), we finally obtain

$$p(0,0) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} (-1)^{m+n} A_{mn} \frac{(\lambda)_m\,(\mu)_n}{m!\,n!}$$

Expressions for the Gegenbauer polynomials [9] are used to calculate the pressure $p(x, y)$ at any point of the contact area.

**4.** Let us clarify the question of the nature of the singularities in the reaction pressure $p(x, y)$ under the stamp upon approaching the boundary and the corners of the contact area. Let (2.8) underlie the analysis. We shall consider that $p_1(x, y)$ contains all the singularities of the function $p(x, y)$, while $p_2(x, y)$ has no singularities (this can be achieved by an appropriate selection of the quantities $\lambda$ and $\mu$). Then the coefficients $A_{mn}$ will tend more rapidly to zero (as $m$ and $n$ increase) and fewer terms in the series in $p_2(x, y)$ will be required to assure a specified accuracy.

The quantities $\lambda$ and $\mu$ in $p_1(x, y)$ evidently depend on the parameter $\varepsilon = b / a$. Considering the limit case $\varepsilon \to 0$, we arrive at the deduction that $\lambda = {}^1/_2$, $\mu = 0$. Similarly, for $\varepsilon \to \infty$ we obtain $\lambda = 0$, $\mu = {}^1/_2$. These conditions can be satisfied if it is assumed that

$$\lambda = {}^1/_2\, e^{-\beta\varepsilon}, \qquad \mu = {}^1/_2 e^{-\beta/\varepsilon} \tag{4.1}$$

where $\beta$ is some constant to be determined below.

Let us clarify the question of the nature of the singularities in the reaction pressure $p(x, y)$ under the stamp as $x \to a$ and $y \to b$, i.e. upon approaching a corner of the contact area. On the basis of (2.8) we have

$$p(x,y) \approx \alpha \left(1 - \frac{x}{a}\right)^{\lambda - 1/2} \left(1 - \frac{y}{b}\right)^{\mu - 1/2} \qquad (x \to a, \quad y \to b)$$

$$\alpha = 2^{\lambda+\mu-1} \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} A_{mn} \frac{(2\lambda)_{2m}\,(2\mu)_{2n}}{(2m)!\,(2n)!}$$

In writing the expression for $\alpha$ it was taken into account that [10]

$$C_n^{\nu}(1) = (2\nu)_n / n!$$

Introducing the new variables $x_1 = 1 - x / a = r\cos\varphi$, $y_1 = 1 - y / b = r\sin\varphi$, we obtain

$$p(r, \varphi) \approx \alpha_0(\varphi) r^{-\psi} \ (r \to 0) \ (\varphi \neq 0, \ \varphi \neq \pi / 2) \tag{4.2}$$
$$\alpha_0(\varphi) = \alpha (\cos \varphi)^{\lambda-1/2} (\sin \varphi)^{\mu-1/2}$$
$$\psi = 1 - \lambda - \mu = 1 - {}^1/_2 (e^{-\beta\varepsilon} + e^{-\beta/\varepsilon})$$

Formula (4.2) establishes the nature of the singularity in the reaction pressure under a rectangular stamp as a corner of the contact area is approached along any line (inclined at an angle $\varphi$ to the $x_1$-axis) under the condition that $0 < x_1 / y_1 < \infty$.

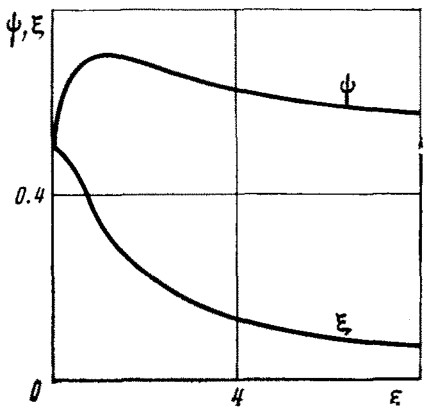


Fig. 2

Using the result of Noble [11], an estimate of the quantity $\psi$ can be given for a square stamp. For $\varepsilon = 1$ we obtain $\psi = 0{,}7$. This result permits estimation of the constant $\beta$ for which we find $\beta \approx 1.204$. Therefore, we can take $\beta = {}^1/_4 e^{\pi/2} = 1.2026$.

Hence, the quantity $\psi$ in (4.2) has been determined completely. A graph of the dependence of $\psi$ on the parameter $\varepsilon$ is presented in Fig. 2.

Let us clarify the nature of the singularity in the reaction pressure $p(x, y)$ under the stamp as $y \to b$, i.e. upon approaching the boundary of the contact area (to the side of the stamp parallel to the $x$-axis). In this case, we have on the basis of (2.8)

$$p(x, y) \approx \bar{\alpha}_1(x) \left(1 - \frac{y}{b}\right)^{-\xi}, \quad \xi = \frac{1}{2} - \mu = \frac{1}{2}(1 - e^{-\beta\varepsilon^{-1}}) \quad (y \to b) \tag{4.3}$$
$$\bar{\alpha}_1(x) = 2^{\mu-1/2} \left(1 - \frac{x^2}{a^2}\right)^{\lambda-1/2} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} A_{mn} \frac{(2\mu)_{2n}}{(2n)!} C_{2m}^{\lambda} \left(\frac{x}{a}\right)$$

Formula (4.3) is valid when $x$ does not tend to $\pm a$. If $x \to a$ and $y \to b$ simultaneously, then (4.2) must be used. A formula similar to (4.3) is used for the case when $x \to a$ (and $y$ does not tend to $b$). A graph of the dependence of $\xi$ on $\varepsilon$ is shown in Fig. 2.

**5.** Let us reduce (3.2) for the coefficients $D_{mnik}$ to a more convenient form for calculations. We perform a change of variables ($x = r\cos\varphi$, $y = r\sin\varphi$), and then we use the Neumann formula twice [10]

$$J_\nu(z) J_\mu(z) = \frac{2}{\pi} \int_0^{\pi/2} J_{\nu+\mu}(2z\cos\theta) \cos[(\mu-\nu)\theta] \, d\theta, \ \mathrm{Re}(\nu+\mu) > -1$$

We obtain

$$D_{mnik} = (-1)^{i+k} \left(\frac{2}{\pi}\right)^2 \int_0^{\pi/2} (\cos\varphi)^{-2\lambda} (\sin\varphi)^{-2\mu} d\varphi \times \tag{5.1}$$
$$\int_0^{\pi/2} \cos[(2n-2k)\theta] \, d\theta \int_0^{\pi/2} \cos[(2m-2i)\psi] \, g(\varphi, \theta, \psi) \, d\psi$$

$$g(\varphi,\theta,\psi)=2^{-2(\lambda+\mu)}\varepsilon^{2\lambda-1}(\lambda+2i)(\mu+2k)\int_0^\infty r^{-2(\lambda+\mu)}\times$$
$$J_{2\lambda+2m+2i}(2\varepsilon^{-1}r\cos\varphi\cos\psi)\,J_{2\mu+2n+2k}(2r\sin\varphi\cos\theta)\,dr$$

The integral in the expression for $g$ $(\varphi, \theta, \psi)$ is evaluated by using the Weber-Schafheitlin integral [12]. We find

$$g(\varphi,\theta,\psi)=\frac{\varepsilon^{2n+2k}}{2^{2\lambda+2\mu+1}}(\lambda+2i)(\mu+2k)\times \tag{5.2}$$
$$\frac{\Gamma(m+n+i+k+{}^1/_2)}{\Gamma(2\mu+2n+2k+1)\,\Gamma(2\lambda+m+i-n-k+{}^1/_2)}\times$$
$$\frac{(\sin\varphi\cos\theta)^{2(\mu+n+k)}}{(\cos\varphi\cos\psi)^{2(n+k-\lambda)+1}}F\left(m+n+i+k+\frac{1}{2},\right.$$
$$\left.n+k-m-i-2\lambda+\frac{1}{2};\quad 2\mu+2n+2k+1;\quad \varepsilon^2\operatorname{tg}^2\varphi\frac{\cos^2\theta}{\cos^2\psi}\right)$$
$$(0<\varepsilon\operatorname{tg}\varphi\cos\theta<\cos\psi)$$
$$g(\varphi,\theta,\psi)=\frac{\varepsilon^{-2m-2i-1}}{2^{2\lambda+2\mu+1}}(\lambda+2i)(\mu+2k)\times$$
$$\frac{\Gamma(m+n+i+k+{}^1/_2)}{\Gamma(2\lambda+2m+2i+1)\,\Gamma(2\mu+n+k-m-i+{}^1/_2)}\times$$
$$\frac{(\cos\varphi\cos\psi)^{2(\lambda+m+i)}}{(\sin\varphi\cos\theta)^{2(m+i-\mu)+1}}F\left(m+n+i+k+\frac{1}{2},\right.$$
$$\left.m+i-n-k-2\mu+\frac{1}{2};\quad 2\lambda+2m+2i+1;\quad \frac{\cos^2\psi}{\varepsilon^2\operatorname{tg}^2\varphi\cos^2\theta}\right)$$
$$(0<\cos\psi<\varepsilon\operatorname{tg}\varphi\cos\theta)$$

The coefficients $D_{mnik}$ defined by (5.1) and (5.2) can now be evaluated by using an electronic computer by replacing the integrals in (5.1) by means of some quadrature formulas. Knowing $D_{mnik}$, we can find an approximate solution of the system of equations (2.7) and can thereby determine the coefficients $X_{mn}$. Then,the depth of impression of the stamp $w_0$ can be found by formula (3.3). By using the relationship (3.1) it is easy to pass to the coefficients $A_{mn}$. The reaction pressure $p$ $(x, y)$ at any point of the contact area can be determined by formula (2.8).

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UDC 539.3

# ON FURTHER DEVELOPMENT OF THE "METHOD OF LARGE $\lambda$" IN THE THEORY OF MIXED PROBLEMS

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The method of large $\lambda$ [1], when the solution of the integral equations is represented as an asymptotic expansion in negative powers of some dimensionless parameter $\lambda$ is used extensively, among the asymptotic methods of investigating the integral equations of the theory of mixed problems. As a rule only several terms of such an asymptotic expansion are constructed successfully.

Certain types of integral equations of the second kind, for which a method is proposed for the construction of all terms of the asymptotic expansion, are investigated below by the method of large $\lambda$. The coefficients and expansions of the required solution in negative powers of $\lambda$ are represented as polynomials in the main argument and recursion formulas are obtained for the coefficients of these polynomials. Considered as examples are the axisymmetric mixed nonstationary problem of heat conduction for a homogeneous half-space and the axisymmetric problem of elasticity theory for the torsion of a truncated sphere by a press.

**1. Solution of the integral equation.** We examine the equation

$$\varphi(t) = \frac{1}{\pi\lambda} \int_{-1}^{1} \varphi(\tau) M\left(\frac{t-\tau}{\lambda}\right) d\tau + g(t) \quad (|t| \leqslant 1) \tag{1.1}$$